How does rainfall become runoff?
A combined tracer and runoff transfer function approach

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Hydrographs are an enticing focus for hydrologic research: they are readily available hydrological data that integrate the variety of terrestrial runoff generation processes and upstream routing. Notwithstanding, new techniques to glean information from the hydrograph are lacking. After early approaches of graphically separating streamflow components, hydrograph separations in the past two decades have focused on tracers as a more objective means to separate the storm hydrograph. These tracer-based methods provide process-based information; however, their implicit assumptions limit their applicability and explanatory power. We present a new method for isotope hydrograph separation that integrates the instantaneous unit hydrograph and embraces the temporal variability of rainfall isotopic composition (one of the largest impediments to the standard use of isotopes as tracers). Our model computes transfer functions for event water and preevent water calculated from a time-variable event water fraction. The transfer function hydrograph separation model (TRANSEP) provides coupled but constrained representations of transport and hydraulic transfer functions, overcoming limitations of other models. We illustrate the utility of TRANSEP by applying it to two rainfall events from a 17 ha catchment at Maimai in New Zealand, where $^{18}$O, rainfall, and runoff data were sampled with a high temporal resolution. We explore which runoff and tracer transfer function (exponential piston flow, gamma distribution, or two parallel linear reservoirs) gave the best results for the proposed model structure and for the example data set. Uncertainty analysis was used to determine if the parameters were identifiable and if the information available for applying TRANSEP was sufficient. The results of the best performing transfer function are considered in detail to identify model performance, illustrate individual event characteristics, and interpret runoff processes in the catchment. INDEX TERMS: 1860 Hydrology: Runoff and streamflow; 1854 Hydrology: Precipitation (3354); 1871 Hydrology: Surface water quality; KEYWORDS: hydrograph separation, transfer function, isotope, instantaneous unit hydrograph


1. Introduction

The processes whereby rainfall becomes runoff continue to be difficult to quantify and conceptualize [McDonnell and Tanaka, 2001; Uhlenbrook et al., 2003]. While much work continues on watershed-scale models of runoff formation, new tools for clear and unambiguous separation and deconvolution of the runoff hydrograph are still beyond our reach. Hydrographs are an enticing focus for hydrologic research: they integrate the variety of upstream routing and watershed flow pathways and are readily available data measured across the globe. Notwithstanding, new ways to read information into the hydrograph are lacking. Early hydrograph-oriented analyses focused on graphical separations of streamflow components (e.g., quick and slow flows) to describe the processes that control the shape, timing, and magnitude of flow reaching the channel [Barnes, 1940; Hewlett and Hibbert, 1967]. A parallel and perhaps more pragmatic track were the development and use of unit hydrograph models. These were developed largely to predict peak discharge in ungauged basins and to provide information about the lumped physical

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characteristics of drainage basins [Sherman, 1932; Clark, 1945]. Both approaches are now well entrenched into hydrologic research and practice [Heerdegen, 1974; Yue and Hashino, 2000].

[1] Since advent of the graphical hydrograph separation, work in the past two decades has focused on the use of tracers as a more objective means to separate the storm hydrograph. Stable isotope hydrograph separations (IHS) [Pinder and Jones, 1969; Sklash et al., 1976] and conservative geo-chemical tracing [Hooper and Shoemaker, 1986] have developed into common tools in small watershed hydrology [Kendall and McDonnell, 1998]. These tracer-based separation approaches have the advantage of providing more process-based information about temporal and geographic sources of runoff. Concurrent with the development of tracer-based hydrograph separations, unit hydrograph approaches have also become more sophisticated. The instantaneous unit hydrograph (IUH) approach [e.g., Dooge, 1959] and the geomorphic unit hydrograph approach of Rodriguez-Iiturbe and Valdes [1979] are now used regularly by research engineers for flood prediction, hydrograph analysis, and flood and reservoir design.

[2] Despite the now common use of IHS and IUH in hydrology, the combination and integration of the two approaches has not yet been explored. Stable isotope mass balance mixing model approaches are somewhat limited in light of the recognized assumptions and limitations implicit in the technique (reviewed by Buttle [1994]). Most problematic is the changing event water composition through a rainfall event, often showing very large monotonic decline with time through the storm [McDonnell et al., 1990; Pionke and DeWalle, 1992; Kendall and McDonnell, 1993]. While McDonnell et al. [1990] have advocated the use of incremental weighting methods to account for the temporal variation and mass tracer allocation, these approaches assume, in effect, instantaneous transfer of event water to the stream and do not incorporate travel time.

[3] We present a new integration of IHS with IUH as a way to quantify this event water transfer more realistically. Our approach builds upon the work of McDonnell et al. [1999] and Weiler et al. [1999], whereby the temporal variability in rainfall isotopic composition is used to model event based age spectra (analogous to the annual time series approach of Małoszewski and Zuber [1982]), to compute event and preevent water contributions to storm runoff. We thus estimate event water residence time distributions for discrete events (building upon Unnikrishna et al. [1995]). In effect, this work is an attempt to combine the process merits of tracer-based hydrograph separation with the hydraulic response of the unit hydrograph in an effort to increase the information gained from the storm hydrograph. Our new method of hydrograph separation embraces the temporal variability of rainfall isotopic composition, but includes new transfer functions for event water and preevent water determined from the time-variable event water fraction. A transfer function representing the runoff response (i.e., the instantaneous unit hydrograph) is used to constrain the event residence time distribution and the hydrograph components. The transfer function approach presented here overcomes many of the limitations of traditional two-component hydrograph separations [Buttle, 1994] and provides separate representations of runoff and tracer responses to storm events that are used to describe hydrologic processes better. While other models [Barnes and Bonell, 1996; Turner and Barnes, 1998] have been developed that use unit hydrograph techniques to represent tracer transport time, they include only a combined transport and hydraulic transfer function or use simple triangular weighting functions [Joerin et al., 2002]. We argue in this paper that both responses are essential to understand catchment behavior, since one response (i.e., the residence time) represents actual conservative solute travel time (i.e., along flow paths) and the other represents hydraulic dynamics (e.g., rainfall-runoff behavior). These responses are typically decoupled with the displacement of preevent water during rainfall periods and the rapid response of new water inputs via well-connected pathways [Bonell, 1998]. Thus the specific objectives of this study are (1) to develop a new lumped-parameter model that combines the transfer of runoff and tracer in a catchment; (2) to test the model and its parameter identifiability for different rainfall events; and (3) to explore how the new model can help the user to understand runoff generation processes better in a catchment.

2. Methods
2.1. Definition of Terms
[6] Many papers in the IHS and IUH literature contain a variety of different terms. We define a number of the terms used in this paper below for clarity of our presentation:

\( g(\tau) \) runoff transfer function (combined tracer and hydraulic responses);

\( h(\tau) \) tracer or particle transfer function (isotopic or solute travel time distribution);

\( h_e(\tau) \) event water transfer function (travel time distribution of new water);

\( h_p(\tau) \) preevent water transfer function (response time distribution of water stored in the catchment prior to a storm event analogous to storage displacement);

\( Q \) total streamflow;

\( Q_e \) event water contribution to streamflow (also referred to as new water);

\( Q_p \) preevent water contribution to streamflow (also referred to as old water).

2.2. Transfer Function Hydrograph Separation Model
[7] The tracer transfer function hydrograph separation model (TRANSEP) is based on the assumption that storm runoff in the stream can be separated into event and preevent components:

\[ Q = Q_p + Q_e \quad (1) \]

\[ CQ = C_p Q_p + C_e Q_e \quad (2) \]

where \( Q \) is the streamflow, \( Q_p \) and \( Q_e \) are the contributions from preevent (i.e., old) and event (i.e., new) water. \( C_p \) and \( C_e \) are conservative tracer (e.g., \( ^{18}O \) or \( ^{2}H \)) concentration values in streamflow, preevent and event water. We assume that the preevent water concentration is constant in space and time for each event. We allow the event water concentration to change with time in our model but rainfall amount and concentration are assumed
to be spatially uniform. The concentration in the stream can then be calculated by combining equations (1) and (2):

\[ C(t) = \frac{Q(t)}{Q(t)} \left( C_e(t) - C_p \right) + C_p \]

The TRANSEP framework is a simple rainfall-runoff model that simulates streamflow by a nonlinear and a linear module (Figure 1), similar to a variety of instantaneous unit hydrograph (IUH) based models [Bras, 1990]. The nonlinear module is the loss function generating an effective precipitation time series [Jakeman and Hornberger, 1993]:

\[ s(t) = b_1 p(t) + (1 - b_2^{-1}) s(t - \Delta t) \]

\[ s(t = 0) = b_3 \]

\[ p_{eff}(t) = p(t) s(t) \]

where \( p_{eff}(t) \) is the effective precipitation, \( s(t) \) is the antecedent precipitation index that is calculated by exponentially weighting the precipitation backward in time according to the parameter \( b_2 \). The parameter \( b_3 \) sets the initial antecedent precipitation index at the beginning of the simulated time series. The parameter \( b_1 \) maintains the water balance (\( \sum p_{eff} = \sum Q \)) over the simulation period and thus can be determined directly from the rainfall-runoff data. The linear module describes a convolution of the effective precipitation and runoff transfer function:

\[ Q(t) = \int_0^t g(\tau) p_{eff}(t - \tau) d\tau \]

where \( g(\tau) \) is the runoff transfer function and thus the rainfall-induced response of catchment runoff.

[5] After the runoff portion of the model is optimized, the runoff transfer function can be used to constrain the hydro-
graph separation, since it represents the combined response of the event and preevent water. On the basis of the rainfall-runoff model, only the effective precipitation can generate streamflow and event water contribution to the stream; therefore the effective precipitation is separated to produce event water and displace preevent water into the stream (Figure 1). This separation can be described by the fraction f that defines the time varying part of precipitation that will eventually reach the stream during the storm as event water runoff.

Various studies using the conventional hydrograph separation approach showed that the event water fraction in the runoff is influenced by the total rainfall amount, the rainfall intensity, and the antecedent wetness conditions [Pionke and DeWalle, 1992; McDonnell et al., 1990; Bottomley et al., 1985]. Therefore our definition of effective precipitation that produces event water runoff, which is defined by the fraction f, should also be a function of these influence factors, as the event water mass is conserved and only transformed by the event water transfer function. Rainfall loss modules in IUH models can successfully calculate effective precipitation by considering rainfall amount, intensity and antecedent wetness conditions. Therefore the loss function generating the effective precipitation by Jakeman and Hornberger [1993] is used to calculate the fraction f as well. The antecedent precipitation index s (equation (4)) is then replaced by the fraction f and b_3 is set to zero as the event water concentration is by definition zero at the beginning of the event.

Similar to the runoff transfer function in the rainfall-runoff module we further assume a time invariant response function of the event water representing the distribution of event water residence times. The event water concentration C_e(t) in the stream can be calculated by [Stewart and McDonnell, 1991; Weiler et al., 1999]:

$$C_e(t) = \frac{\int_0^t C_e(t-\tau) p_{eff}(\tau)f(\tau-h_e(\tau))d\tau}{\int_0^t p_{eff}(\tau)f(\tau-h_e(\tau))d\tau}$$

(6)

where \(C_e\) is the concentration in the rainfall, which can be varying over time, \(f\) is the fraction of effective precipitation that becomes event water (i.e., “new” water), and \(h_e(\tau)\) is the transfer function of the event water (i.e., the residence time distribution). The advantage of the modified convolution (equation (6)) is that it allows for direct weighting of the input concentrations as opposed to a predefined weighting relationship [e.g., Maloszewski et al., 1992]. The denominator then defines the event water runoff in the stream by:

$$Q_e(t) = \int_0^t p_{eff}(\tau)f(\tau-h_e(\tau))d\tau$$

(7)

In contrast to previous approaches, the weighting of the precipitation that generates event water in the stream is calculated based on the effective precipitation and not the gross precipitation. This approach (equation (6)) also allows for a time varying event water fraction, which is in contrast to other approaches [McDonnell et al., 1999].

Combining equations (5) and (7), the stream event water fraction \(X\) is defined by:

$$X(t) = \frac{Q_e(t)}{Q(t)} = \frac{\int_0^t p_{eff}(\tau)f(\tau-h_e(\tau))d\tau}{\int_0^t p_{eff}(\tau)g(\tau)d\tau}$$

(8)

And finally the concentration in the stream can be derived by inserting equations (5) and (6) into equation (3):

$$C(t) = \frac{1}{Q(t)} \int_0^t C_e(t-\tau) p_{eff}(\tau)f(\tau-h_e(\tau))d\tau - \frac{C_p}{Q(t)} \int_0^t p_{eff}(\tau)f(\tau-h_e(\tau))d\tau + C_p$$

(9)

This equation can then be used to simulate the streamflow concentration, if the effective precipitation and simulated streamflow are determined a priori. Therefore it is necessary first to optimize the rainfall-runoff model to the measured streamflow and then the event water transfer module to the measured concentration in the stream (Figure 1). Likewise, the preevent water runoff can be calculated from the total streamflow and the simulated event water runoff (equation (7)). Then the preevent water transfer function \(h_p(\tau)\) can be directly calculated from the runoff transfer function \(g(\tau)\) and the event water transfer function \(h_e(\tau)\):

$$h_p(\tau) = \frac{1}{1-f} \left[ g(\tau) - f h_e(\tau) \right]$$

(11)

However, because of the assumption that \(f\) is changing with time, \(h_p(\tau)\) has to be optimized. There are many potential transfer functions for hydrological applications. In the IUH literature, probability distributions with two to three parameter models (gamma, lognormal) and linear reservoir approaches are used [Vieseman et al., 1989; Shamseldin and Nash, 1998]. The linear reservoirs are arranged in series or parallel. In the tracer and solute transport literature, the convection dispersion equation (CDE), the lognormal probability distribution [Jary and Roth, 1990], the exponential and piston flow model [Maloszewski and Zuber, 1982], and the gamma distribution [Kirchner et al., 2000] have been widely used.

In order to make TRANSEP flexible and to test multiple transfer function approaches, we implemented three different models for defining the runoff, event water and prevent water transfer functions.

1. Exponential-piston flow (EPM)

$$h(\tau) = g(\tau) = \frac{n}{\tau_0} \exp \left( \frac{-\eta \tau}{\tau_0} + \eta - 1 \right) \quad \text{for} \quad \tau \geq \tau_0 (1 - \eta^{-1})$$

(12a)

$$h(\tau) = g(\tau) = 0 \quad \text{for} \quad \tau < \tau_0 (1 - \eta^{-1})$$

(12b)
where $\tau_0$ is the mean residence time and $\eta$ is the parameter which equals the total volume of water divided by the exponential flow volume. Thus the model is equal to the exponential distribution or a simple linear well-mixed reservoir when $\eta = 1$ [Maloszewski and Zuber, 1982].

2. Gamma distribution or linear reservoirs in series

$$h(\tau) = g(\tau) = \frac{1}{\beta T(\alpha)} \exp\left(\frac{-\tau}{\alpha}\right)$$

where $\alpha$ is the shape parameter, $\beta$ is the scale parameter and the mean residence time is given by $\beta \alpha$.

3. Two parallel linear reservoirs (TPLR)

$$h(\tau) = g(\tau) = \frac{1}{\tau_f} \exp\left(-\frac{\tau}{\tau_f}\right) + \frac{1 - \phi}{\tau_s} \exp\left(-\frac{\tau}{\tau_s}\right)$$

where $\tau_f$ and $\tau_s$ are the mean residence times of the fast and slow responding reservoirs, respectively. The parameter $\phi$ defines the partition of the input into the fast responding reservoir. Similar to equation (12), we can also define a parameter $\eta$, which shifts the transfer function and thus implies a time delay. Since the event and pre-event water transfer function is coupled to the runoff transfer function, the models describing those transfer functions are the same.

Depending on the chosen transfer function, five to seven parameters have to be optimized within the rainfall-runoff model. We used ant colony optimization (ACO) to solve the inverse estimation problem of the unknown parameters [Abbaspour et al., 2001]. It was shown that this technique efficiently finds the optimum solution for a wide range of applications. After optimizing the parameters describing the runoff response, the two parameters describing the fraction of effective precipitation that produces event water runoff and the two to three parameters describing the event water transfer function were optimized. Finally, the two to three parameters describing the pre-event water transfer function were optimized. This stepwise optimization technique ensures that the inverse problem is not ill posed and that the parameters are identifiable.

The selection of the goodness-of-fit measures further influences the optimization results [Beven, 2000]. For fitting hydrological models to discharge data, the model efficiency suggested by Nash and Sutcliffe [1970] has become very popular and is suggested for optimization of rainfall-runoff models. As recommended by Legates and McCabe [1999] we also evaluated model error using the root mean square error (RMSE), which preserves the simulation units as opposed to relative error measures such as efficiency. We finally used the average of the model efficiency and $(1 - \text{RMSE})$ to optimize TRANSEP.

3. Application to Field Data

3.1. Example Data Set

Two rainfall events from the 17 ha K catchment at Maimai in New Zealand [McGlynn, 2002; McGlynn et al., 2002; McGlynn and McDonnell, 2003b] for full site description were utilized to demonstrate application of the TRANSEP model. The K catchment hillslopes are steep (average 34 degrees), short (100 – 150 meters), and composed of regular intervals of spurs and hollows. The soils are shallow (~1 m), are highly permeable (saturated-hydraulic conductivity = 250 mm h$^{-1}$), and are underlain by a poorly permeable, firmly compacted, moderately weathered, early Pleistocene conglomerate. Streamflow was determined at five-minute intervals from stream stage measured at the K catchment outlet with a 90° V notch weir. Rainfall was measured in 0.2 mm increments with a tipping bucket rain gauge. Precipitation samples were collected in 5 mm increments with a sequential rainfall sampler [Kennedy et al., 1979]. Streamflow was sampled both manually and with an ISCO automated sampler at one-hour intervals. All samples were analyzed for $\delta^{18}O$ at the USGS Stable Isotope Laboratory in Menlo Park, CA by mass spectrometer and reported in % relative to VSMOW with 0.05‰ precision. We linearly interpolated the data to a time step of 30 min for applying TRANSEP in order to capture short-term fluctuations in the rainfall, runoff and concentration signal.

We intensively monitored two discrete rainfall events. Event 1 was 27 mm of rainfall under low antecedent moisture conditions for this site ($AP_{14} = 17$ mm, and $AP_{17} = 7$ mm), resulting in 5.2 mm of runoff and a runoff ratio ($Q/P$) of 0.19. Event 2 was 70 mm under high antecedent moisture conditions ($AP_{14} = 44$ mm, and $AP_{17} = 34$ mm), and resulted in a runoff ratio of 0.52. Base flow $\delta^{18}O$ prior to both events was consistent ($\pm$0.5 per mil) with pre-event $\delta^{18}O$ measured in riparian and hillslope positions in the K catchment (as sampled from wells and suction lysimeters). Therefore stream base flow was used as the pre-event water signature in the TRANSEP model. We allowed the event water (input) concentration to change with time in our model, in accordance with $^{18}O$ determined by discrete sampling and analysis of each sequential 5 mm of precipitation. Base flow was subtracted using a constant value of flow from the beginning of the event since the unit hydrograph portion of the model only simulates direct runoff.

3.2. Transfer Functions

Because of the variety of transfer functions that were used for describing water and tracer response in the catchment, it is necessary to explore which transfer function gives the best results for the proposed model structure and for the example data set. The three different transfer functions (EPM, gamma distribution, and 2 parallel linear reservoirs) were used to optimize the model for the two selected events. The model performance summarized in Table 1 for the different realizations shows that the 2 parallel linear reservoir (TPLR) transfer function generally performs better than the other two transfer functions (EPM and gamma distribution). Comparing solely the EPM and gamma transfer functions, the performance of the EPM is generally better for predicting the concentration and the gamma distribution is better for predicting the streamflow. The second improvement is directly related to the sequential parameter optimization (first rainfall-runoff, then concentration) where a better fit for the runoff data increases the performance of the tracer concentration optimization. The general improvement can also be attributed to the increase of parameters (EPM and gamma equal two parameters, TPLR equals four parameters). However, a visual control of the simulation results revealed the worse performance of the EPM and gamma transfer function for the recession part of the hydrograph and the isotope concentration, thus indicating that the simpler two-parameter transfer functions...
cannot capture the complex runoff generation processes in the studied catchments, where evidently, a fast and a slow component are responsible for generating runoff. Therefore, the TPLR transfer function was favored not only because of the better model performance but also in terms of capturing the runoff generation processes in the catchment.

### 3.3. Analysis of Final Results

The results of the best performing transfer function using the two parallel linear reservoirs was considered in detail to identify its performance and to point out the individual characteristics for each event. This example is also used to explain the individual results of TRANSEP. Figure 2 compares the simulation results with the measurements for the two events. The smooth single peaked hydrograph of event 1 could not be reproduced in detail, since the rising and falling limbs were over predicted and the peak was underestimated. For event 2, the peak flow is captured quite well; however, a small time lag between the observed and simulated streamflow resulted in an overestimation in the first part of the falling limb. The second part of the falling limb, however, was underestimated. Despite these imperfect performances of the rainfall-runoff module, the simulated $^{18}$O concentration in the stream was well characterized for both events. The runoff simulations cannot be expected to perform well in all cases due to the model structure simplicity and linearity assumption. The peak concentrations in $^{18}$O, as well as the different recession characteristics of the two events were well reproduced. Finally, the standardized residuals of the streamflow, which were defined by dividing the residual by the root mean square error, were compared with the standardized residuals of the $^{18}$O concentrations for the two events in Figure 2 to analyze a potential correlation between the residuals of streamflow and concentration. For event 1, there seems no correlation of the residuals; however, for event 2, there is a small correlation between the two residuals on the falling limb of the hydrograph. Generally, the serial correlation is weak, the error variance is homoscedastic, and the simulations appear to be relatively independent.

The simulation results of event water contribution from the rainfall and in the stream are shown in Figure 3. The effective precipitation and the proportion that becomes event water are shown for the two events in the upper panel. The rainfall that becomes event water is low at the beginning and is mostly dependent on the intensity of the effective precipitation. This behavior can be observed for both events, despite marked differences in the magnitude of the events. The resulting streamflow and event water runoff are shown in the middle panel of Figure 3. The event water is very low at the beginning and in the second part of the recession, but event water contributes significantly during the peak runoff. The actual fraction of event water in the rainfall that becomes event water in the stream ($f$) and the fraction of event water in the stream ($X$) are shown in the bottom panel of Figure 3. During the maximum effective rainfall intensity, and consequently during peak runoff, event water contributes between 35% and 40% of the total runoff. The fraction of event water in the rainfall reaches a maximum during the highest rainfall intensity signal and decreases very quickly. The streamflow shows a distinct event water recession with a steady reduction of the event water fraction for event 1 and a rapid reduction with a late second peak for event 2.

Each estimated transfer function is shown in Figure 4. For each event, the runoff transfer function $g(t)$, the event water $h_e(t)$ and preevent water $h_p(t)$ transfer function are compared. For event 1, the three transfer functions are almost identical. The event water transfer function shows a slightly larger contribution at early times compared to the hydraulic transfer function, resulting in a preevent water with lower contribution at early times. For event 2, the event water transfer function shows a significantly higher peak and a faster decline compared to the hydraulic transfer function. The preevent water transfer function consequentially is more damped than the hydraulic transfer function. Comparing the two events, event 1 shows a damped and lagged transfer function, whereas event 2 is skewed heavily toward early times and rapid response. The two events become more distinct when the same transfer functions are plotted on a logarithmic scale (bottom panel of Figure 4). For event 1, all transfer functions show a linear decline in the log space, which means that a simpler transfer function (single linear reservoir) may be sufficient to describe the system. This observation is shown in Table 1, where the model performances of event 1 for the EPM are reasonable; thus the more simple transfer function describes the system response well. For event 2, there is a distinct break in the decline of the transfer functions. This break occurs at $\sim$9 hours for the runoff transfer function, at $\sim$6 hours for the event water transfer function, and at $\sim$12 hours for the preevent water transfer function. These breaks can be explained by two distinct reservoirs transferring the effective precipitation into the stream. This behavior also explains the poor performance of the EPM and gamma transfer function for the second event (Table 1).

### 3.4. Parameter Identifiability

The GLUE methodology [Freer et al., 1996] was applied to determine the identifiability of the individual parameters used in TRANSEP. 10,000 Monte Carlo realizations were simulated and the randomly chosen parameter

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**Table 1. Model Performance for the Different Transfer Functions**

<table>
<thead>
<tr>
<th>Transfer Function</th>
<th>Event</th>
<th>Efficiency</th>
<th>RMSE, mm/h</th>
<th>Efficiency</th>
<th>RMSE, %</th>
<th>Event Water Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPM</td>
<td>1</td>
<td>0.77</td>
<td>0.07</td>
<td>0.95</td>
<td>0.06</td>
<td>0.22</td>
</tr>
<tr>
<td>EPM</td>
<td>2</td>
<td>0.86</td>
<td>0.25</td>
<td>0.71</td>
<td>0.13</td>
<td>0.20</td>
</tr>
<tr>
<td>Gamma distribution</td>
<td>1</td>
<td>0.87</td>
<td>0.05</td>
<td>0.78</td>
<td>0.13</td>
<td>0.23</td>
</tr>
<tr>
<td>Gamma distribution</td>
<td>2</td>
<td>0.96</td>
<td>0.14</td>
<td>0.57</td>
<td>0.16</td>
<td>0.22</td>
</tr>
<tr>
<td>Two parallel linear reservoirs (TPLR)</td>
<td>1</td>
<td>0.94</td>
<td>0.03</td>
<td>0.92</td>
<td>0.08</td>
<td>0.24</td>
</tr>
<tr>
<td>Two parallel linear reservoirs (TPLR)</td>
<td>2</td>
<td>0.96</td>
<td>0.13</td>
<td>0.86</td>
<td>0.09</td>
<td>0.18</td>
</tr>
</tbody>
</table>
values were plotted against the selected objective function for each parameter as dotty plots (Figure 5). While both events were examined for parameter identifiability, this paper describes only the event 2 analysis given the similar identifiability results for both events. The best fit parameter values using the ant colony optimization are highlighted by a thick dot. The selected likelihood value is an equally weighted combination of the model efficiency and $1 - \text{RMSE}$, where a value of 1 would mean optimal performance of the model. The dotty plots can be used to indicate whether there is only a small range of parameter values that can give good results or whether the whole set of parameter values can give good results. This then provides a measure of the parameter identifiability and indicates whether multiple parameter sets can yield the same well-fit simulation (i.e., equifinality).

Figure 2. Optimization results for the two events (left column for event 1, right column for event 2) showing streamflow $Q$, $^{18}\text{O}$ concentration in the stream $C$, and the standardized residuals for discharge and concentration.
For the rainfall-runoff module, the three parameters defining the loss function generating the effective precipitation time series ($b_1$, $b_2$, $b_3$) show a low sensitivity, with $b_1$ and $b_2$ showing at least some constraint. These poor constraints are probably related to the short simulation time and the use of only one single event. Three of the four parameters defining the runoff transfer function ($t_f$, $f$, $h$) are much better identified. However, the parameter $\tau_s$ that defines the mean response time of the slow reacting reservoir cannot be identified for this relatively short event. It is likely that a longer time series would increase the identifiability of the mean response time of the slow reacting reservoir. However, the poor identifiability of some parameters, mainly parameters of the loss function, is a
problem with all conceptual models in hydrology where it is extremely difficult to have all parameters well identified. We present the identifiability information in Figure 5 as a way to honestly evaluate the model performance and to interpret parameter sensitivity.

For the event water transfer module, only two parameters \( b_1, b_2 \) depicted the constraint for the fraction of effective precipitation that becomes event water \( f \) as the parameter \( b_3 \) is zero by definition. The parameter defining the total fraction of event water \( b_1 \) is well defined because the concentration change in the streamflow is constrained by the water volume. The second parameter \( b_2 \) defining the backward weighting of the effective precipitation is poorly identified. Parameters for the event water transfer function were reasonably well defined. The mean response time of the fast reacting reservoir \( t_f \) and the partition coefficient \( \phi \), are very well identified; however, the mean response time of the slow reacting reservoir \( t_s \) is poorly identified. The poor identifiability of \( t_s \) is probably related to the short simulation time and the single recession, as it was for the runoff transfer function. These results indicate that the information available in the rainfall and stream \( ^{18} \text{O} \) concentration time series are sufficient to define a transfer function for the event water. We can also assume that the sequential parameter optimization (first rainfall-runoff, then concentration) increases the identifiability of the six parameters defining the separation and transfer of the event water.

4. Discussion

4.1. New TRANSEP Approach

A review on the state of forest and catchment hydrology by Bonell [1993] concluded that “more field experiments, coupled with laboratory work, on the lines of McDonnell [1990] are urgently required”. Despite the many studies that have done this in the past decade by combining tracer and hydrometric rainfall-runoff data, we still do not well understand the timing, flow path, and source behavior of catchments [Burns, 2002]. One reason is that we still lack the tools necessary to extract the process-level information from these new combined tracer-hydrometric data sets that include event-based isotope and discharge data. Numerous recent studies have shown the decoupled nature of hydraulic response and tracer transport associated with pre-event water displacement during rainfall periods and rapid transfer of new water inputs to the stream. TRANSEP is a quantitative approach to describe the residence time of solute transport and transmittance of hydraulic behavior to help understand, as Kirchner [2003] notes, “the often paradox-
ical relationship between pre-event and event water delivery to streams.''

By using separate transfer functions that describe the travel times of event and pre-event water and the overall water flux response, we argue that TRANSEP can improve the understanding of runoff generation processes in catchments where it is applied. Previous approaches that combine water flux and solute transport [e.g., Barnes and Bonell, 1996] are not able to separate and quantify processes like displacement of pre-event water and preferential flow contribution of event water since both are incorporated into the same function. In addition, Joerin et al. [2002] suggested that influence functions (i.e., transfer functions) might improve uncertainty in hydrograph separations. TRANSEP provides new and additional analytical powers by embracing the temporal variation of rainfall tracer composition (often a key limitation to standard isotope based hydrograph separation approaches) to determine more accurately the hydrograph components. Second, the crossover effect of the rainfall and streamflow tracer signals that are often observed [Buttle, 1994] do not negatively affect the separation because the model transfers mass, not concentration. Thus the number of storms in a given data set to which TRANSEP can be applied is large relative to other existing models. TRANSEP provides coupled, but constrained, representations of transport and hydraulic transfer functions and provides a new way forward to the now standard tool of two component isotope hydrograph mass balance separations.

### 4.2. TRANSEP Comparison to Two-Component Hydrograph Separations

The TRANSEP model, using the two parallel linear reservoirs (TPLR) transfer function, calculated 24% and 18% event water in storms 1 and 2, respectively. Runoff from the Maimai K catchment was also separated into its event and pre-event water components based on traditional two-component hydrograph separation methods (equation (3)). The rainfall or event water component was weighted based on the incremental mean weighting method [McDonnell, 1990]. We found that 27% of the runoff in event 1 and 29% of the runoff in event 2 was event water, despite a seven-fold increase in total runoff from event 1 to event 2 (B. L. McGlynn et al., Role of catchment scale in runoff generation, submitted to Water Resources Research, 2003) (hereinafter referred to as McGlynn et al., submitted manuscript, 2003). At peak runoff, event water fractions of 36% in event 1 and 37% in event 2 were calculated. These results correspond to

![Figure 5](image-url)

**Figure 5.** Dotty plots of the equally weighted combination of the model efficiency and \((1 - \text{RMSE})\) for the major parameters using the GLUE approach for event 2. The thick dots highlight the optimized value for each parameter.
the 32% and 38% new water at peak runoff calculated by the TRANSEP model.

[30] The most marked difference between the traditional two-component model and TRANSEP was the total event water runoff calculated for storm 2. The 11% difference in new water proportion is likely due the introduction of effective precipitation in the TRANSEP approach, one that more realistically portrays rainfall influence on hydrograph components [Genereux, 1998]. Early in event 2, rainfall $^{18}$O was similar to base flow $^{18}$O for the first 10 mm of precipitation. The closer the event and preevent components (or end-members) are to one another, the higher the proportion of event water necessary to explain deflection from base flow. In the traditional two-component separation, this rainfall was weighted heavily early in the event and continued to influence the running mean rainfall $^{18}$O throughout the event, resulting in less separation between the preevent water and event water signatures early in the event and correspondingly higher total event water estimates. The TRANSEP model, in contrast, constrains the rainfall with the effective precipitation weighting (equation (4c)), thus weighting early event precipitation less (Figure 3). We argue that the effective precipitation weighting for rainfall $^{18}$O produces a more realistic input function for event rainfall and correspondingly calculates less event water.

4.3. TRANSEP and the Isolation of Tracer and Runoff Response

[31] Conservative tracer signatures (e.g., $^{18}$O) integrate water molecule transport and mixing while the runoff hydrograph response represents both hydraulic or pressure response to precipitation and water transport. Quantifying and understanding both responses is essential to understanding catchment behavior and runoff generation mechanisms and controls. However, these two processes are typically decoupled. Each step in the runoff generation process results in deviation between the tracer response and the hydraulic response. For example, McGlynn and McDonnell [2003a] found that infiltration of event water displaced and mixed with preevent water on a trenched and gauged hillslope at Maimai, resulting in a dynamic hillslope hydrograph response at the base of the hillslope with little observed new water. Another complication is the spatial variability of runoff generation and travel times to the catchment outlet for event, preevent, and runoff response: each can be generated in different spatial locations in a catchment (McGlynn et al., submitted manuscript, 2003). In addition, tracer transport is slower than hydraulic wave propagation down the channel network. Once runoff enters the stream channel network, additional decoupling is possible as pressure propagation velocities exceed particle velocities and tracer is held back in transient storage exchange.

[32] The runoff response is a summation of the event water response and the preevent water response. Essentially, the event water landing on the catchment initiates the runoff generation process. Event water can be partitioned into (1) loss (i.e., soil moisture recharge or storage, and evapotranspiration) as modeled in the effective precipitation calculation in TRANSEP, (2) preevent water displacement, and (3) remaining event water response. TRANSEP generates transfer functions and residence time distributions to deconvolute the storm hydrograph into each of the three portions. If the calculated three transfer functions are very similar, it is likely that runoff was generated by a well-mixed or connected event water and preevent water system during the runoff response (Figure 4). This simulated behavior can be related to the actual observed runoff processes during event 1: runoff was generated primarily in the valley bottom riparian zones [McGlynn and McDonnell, 2003a]. For event 2, the three transfer functions are different, indicating either a poorly mixed flow system or, perhaps more realistically for this event, runoff generation from different areas (riparian zone and hillslope) with different flow pathways or different response dynamics. The relative shapes of the runoff transfer function, event water transfer function, and preevent water transfer function between events 1 and 2, and their inter-comparison, provide insight into likely catchment runoff processes. The linear nature of each transfer function as plotted in log space (Figure 4) shows that one reservoir could be used to characterize the transfer function in event 1. In event 2, however, runoff was generated on hillslopes throughout the catchment in addition to the valley bottom riparian zone. As a result, in log space, the transfer functions are nonlinear and cannot be adequately represented by one reservoir: two linear reservoirs in parallel are required.

4.4. Future Research With TRANSEP

[33] While this paper presents the new TRANSEP model and an initial application, we argue that there are many potential uses of the approach to hydrological studies. For instance, TRANSEP could be used to analyze the results from one or more rainfall events within different sized watersheds. The model and its resulting runoff and tracer transfer functions and their parameterizations could then be used to study the scaling behavior of the residence time of water in the watersheds in a much more efficient and straightforward way than is possible with existing techniques. We also expect that extension of the model for application with other tracers, and even reactive solutes, could be possible whereby the residence time distribution of certain solutes or nutrients in the watershed could be quantified. In this approach one would have to describe the mobilization process due to the precipitation or snowmelt events. Because the description of the TRANSEP loss function for calculating the effective rainfall and the event water contribution is based on the function introduced by Jakeman and Hornberger [1993] for long term unit hydrograph studies, we hope to use TRANSEP for longer time-scale simulations such as by Pinault et al. [2001]. Seasonal changes of the prevent water concentrations (as observed in many experimental studies) could then be introduced. Again, because TRANSEP describes the runoff and tracer response in a catchment by a lumped transfer function approach, there is potential to use it as a change detection tool. We envisage application to studies of land use change, forest fire, and climate change where hitherto changes have not been be clearly detectable in the runoff response. Because TRANSEP can detect changes in the runoff, event water transfer, and preevent water transfer, changes in the runoff generation processes might also be detected.

5. Conclusion

[34] We developed the new tracer transfer function hydrograph separation model TRANSEP that builds on the simple but integrated concepts of the instantaneous unit hydrograph
(IUH) and isotope hydrograph separations (IHS). TRANSEP uses water flux and isotopic data from precipitation and streamflow to derive transfer functions of runoff, event and pre-event water by capitalizing on the temporal variation of rainfall tracer composition. A two-step optimization procedure significantly increased the identifiability of the parameters defining the transfer functions and the nonlinear partitioning of rainfall into effective precipitation and event water contribution to stream runoff. Comparing different transfer functions, we found the two parallel linear reservoirs (TPLR) transfer function more suitable because of the better model performance but also because of the way it captures the runoff generation processes in the catchment. TRANSEP thus provides coupled, but constrained representations of transport and hydraulic transfer functions and provides a new way forward to the now standard tool of two component isotope hydrograph mass balance separations. It infuses information into the IUH by the combination of runoff and event/pre-event water transfer, thus enabling one to identify runoff generation processes in a catchment.

References


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